Load Frequency Control of a Four-Area Power System Using Linear Quadratic Regulator

Yogendra Arya[1], Narendra Kumar[2], S.K. Gupta[3]

[1]Department of Electrical and Electronics Engineering, Maharaja Surajmal Institute of Technology, New Delhi, India

[2]Department of Electrical Engineering, Delhi Technological, University, New Delhi, India

[3]Department of Electrical Engineering, Deenbandhu Chhotu, Ram University of Science & Technology, Haryana, India

y_arya40@yahoo.com

***Abstract*- In this paper an optimal integral controller is proposed for load frequency control problem of multi-area electrical power system. The design of optimal controller uses modern optimal control theory. A state model of a four-area interconnected power system is developed. Dynamic analyses have been done without controller, with integral controller and with optimal integral controllers using Matlab Simulink-Workspace. A comparison between some recent trend controllers and proposed optimal integral controller is presented further and it has been shown that the performance of the proposed controller is superior in terms of overshoot and settling time.**

Keywords-Optimal Control; Automatic Generation Control (AGC); Four-Area Power System; Load Frequency Control (LFC); Integral Controller; Area Control Error (ACE)

I. INTRODUCTION

Large-scale power systems are normally composed of control areas or regions representing coherent groups of generators. Various areas are interconnected through tie-lines. Tie-lines are utilized for contractual energy exchange and provide inter-area support in case of abnormal conditions. Real and reactive power generations must change accordingly to match the load perturbations. Load frequency control is essential for successful operation of power systems to maintain frequency within the limited band [1]. To accomplish this, it becomes necessary to automatically regulate the operations of main steam valves or hydro gates in accordance with a suitable control strategy, which in turn controls the real power output of electric generators. The controlling of the output of electric generators in this way is termed as automatic generation control (AGC) [2]. Automatic generation control is the regulation of power output of controllable generators within a prescribed area in response to change in system frequency, tie-line loading, or a relation of these to each other, so as to maintain the schedule system frequency and establish interchange with other areas within predetermined limits. AGC can be sub-divided into primary and secondary control modes. The overall performance of AGC in any power system depends on the proper design of both primary and secondary control loops. Secondary controllers are designed to regulate the area control errors to zero effectively [2]. So the overall performance of AGC in any power system depends on the proper design of both primary control loop (selection of R) and secondary control loops (selection of gain for supplementary controller). A control strategy is needed that not only maintains constancy of frequency and desired tie-power flow, but also achieves zero steady state error and inadvertent interchange. Among the various types of load frequency controllers, the most widely employed are integral (I), proportional plus integral (PI) and proportional plus integral plus derivative (PID) controllers. The use of modern optimal control theory can improve the dynamic performance of these controllers [3]. Over the past decade many control strategies for load frequency control of power systems such as linear feedback [4], variable structure control [5] etc. have been proposed in order to improve the transient response. An adaptive controller with self-adjusting gain setting was proposed for LFC in [6]. Artificial Neural Networks have been successfully applied to the LFC problem [7], [8]. Moreover fuzzy logic control techniques for LFC problem are mostly based on fuzzy gain scheduling of integral [9] and proportional integral (PI) [10], [11], [12] controller parameters. Recently hybrid neuro fuzzy control strategy is proposed for LFC to improve performance of the fuzzy controller [13]. Recently many researchers have applied Genetic algorithm [14] and neural networks [15] to controllers design to improve the dynamic performance of the power system. In a control schemes LFC by automatic design of the fuzzy rules in the fuzzy gain scheduling control approach by genetic algorithm is proposed in [16].

This paper uses the optimal control strategy to optimize integral controller for four-area interconnected power system. Simulation results shows that the proposed controller in terms of dynamic performance such as peak overshoot, settling time and integral absolute error is better than integral controller and many intelligent controllers used recently.

II. FOUR-AREA POWER SYSTEM

An interconnected power system is divided into control areas connected by tie lines. A four-area interconnected power system (Fig.1) of a non-reheat thermal plant is used to explain the motivation of the proposed method. The frequency deviation in all areas severely affects the production and the quality of frequency sensitive industries such as the spinning and weaving industry, petrochemical industry, pulp and paper industry, etc. Furthermore, the lifetime of machine apparatuses on the load side will be reduced. The tie-line power flow and frequency of the area are affected by the load changes. Therefore, it can be considered that each area needs its system frequency and tie-line power flow to be controlled.

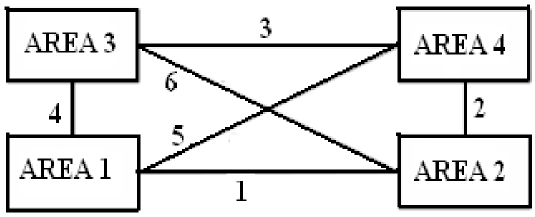


Fig. 1 Four interconnected control areas (six tie lines)

III. UNCONTROLLED FOUR-AREA POWER SYSTEM

A four-area power system connected by six tie lines illustrated in Fig.1 is taken as a test system in the study. Each area decides steam turbine contains a governor and a generator.

The block diagram for controlled system given in Fig.2 is also used for uncontrolled system. The blocks for integral controller i. e. 19-22 and frequency sensor i. e. b_1-b_4 given in Fig.2 are neglected to formulate uncontrolled system's state model by writing the differential equations describing each individual block in terms of state variables.

Mathematical Model Development using Figure 2

$$x_1 = \Delta f_1 \quad x_2 = \Delta P_{g1} = \Delta P_{t1} \quad x_3 = \Delta Y_{E1}$$
$$x_4 = \Delta f_2 \quad x_5 = \Delta P_{g2} = \Delta P_{t2} \quad x_6 = \Delta Y_{E2}$$
$$x_7 = \Delta f_3 \quad x_8 = \Delta P_{g3} = \Delta P_{t3} \quad x_9 = \Delta Y_{E3}$$
$$x_{10} = \Delta f_4 \quad x_{11} = \Delta P_{g4} = \Delta P_{t4} \quad x_{12} = \Delta Y_{E4}$$
$$x_{13} = \Delta P_{tie,\,12} \text{ or } \Delta P_{tie,\,21} \quad x_{16} = \Delta P_{tie,\,34} \text{ or } \Delta P_{tie,\,43}$$
$$x_{14} = \Delta P_{tie,\,31} \text{ or } \Delta P_{tie,\,13} \quad x_{17} = \Delta P_{tie,\,32} \text{ or } \Delta P_{tie,\,23}$$
$$x_{15} = \Delta P_{tie,\,41} \text{ or } \Delta P_{tie,\,14} \quad x_{18} = \Delta P_{tie,\,42} \text{ or } \Delta P_{tie,\,24}$$
$$w_1 = \Delta P_{d1} \quad w_2 = \Delta P_{d2} \quad w_3 = \Delta P_{d3} \quad w_4 = \Delta P_{d4}$$

From block 1,

$$\frac{x_1}{x_2 - w_1 + a_{31}x_{14} + a_{41}x_{15} - x_{13}} = \frac{K_{ps1}}{1 + T_{ps1}s}$$

$$\dot{x}_1 = -\frac{1}{T_{ps1}}x_1 + \frac{K_{ps1}}{T_{ps1}}x_2 - \frac{K_{ps1}}{T_{ps1}}x_{13} + \frac{a_{31}K_{ps1}}{T_{ps1}}x_{14} + \frac{a_{41}K_{ps1}}{T_{ps1}}x_{15} - \frac{K_{ps1}}{T_{ps1}}w_1 \quad (1)$$

Similarly,

$$\dot{x}_2 = -\frac{1}{T_{t1}}x_2 + \frac{1}{T_{t1}}x_3 \quad (2)$$

$$\dot{x}_3 = -\frac{1}{R_1T_{sg1}}x_1 - \frac{1}{T_{sg1}}x_3 + \frac{1}{T_{sg1}}x_{13} - \frac{a_{31}}{T_{sg1}}x_{14} - \frac{a_{41}}{T_{sg1}}x_{15} \quad (3)$$

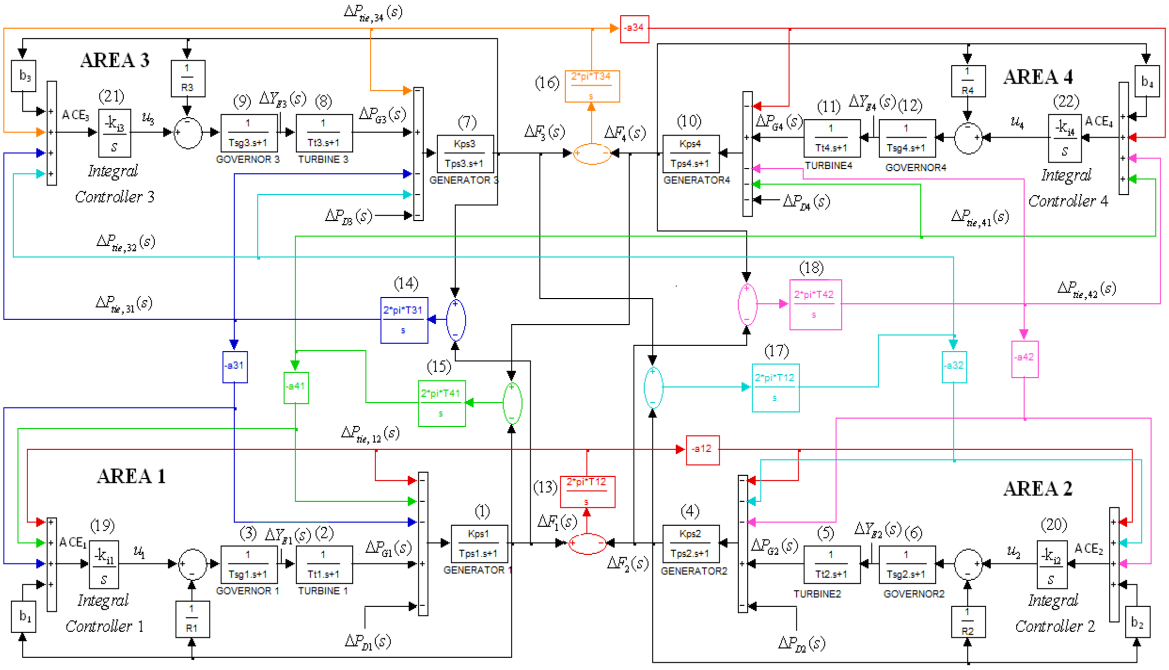


Fig. 2 Block diagram of four-area power system with integral controller

$$\dot{x}_4 = -\frac{1}{T_{ps2}}x_4 + \frac{K_{ps2}}{T_{ps2}}x_5 + \frac{a_{12}K_{ps2}}{T_{ps2}}x_{13} + \frac{a_{32}K_{ps2}}{T_{ps2}}x_{17} + \frac{a_{42}K_{ps2}}{T_{ps2}}x_{18} - \frac{K_{ps2}}{T_{ps2}}w_2 \quad (4)$$

$$\dot{x}_5 = -\frac{1}{T_{t2}}x_5 + \frac{1}{T_{t2}}x_6 \quad (5)$$

$$\dot{x}_6 = -\frac{1}{R_2T_{sg2}}x_4 - \frac{1}{T_{sg2}}x_6 - \frac{a_{12}}{T_{sg2}}x_{13} - \frac{a_{32}}{T_{sg2}}x_{17} - \frac{a_{42}}{T_{sg2}}x_{18} \quad (6)$$

$$\dot{x}_7 = -\frac{1}{T_{ps3}}x_7 + \frac{K_{ps3}}{T_{ps3}}x_8 - \frac{K_{ps3}}{T_{ps3}}x_{14} - \frac{K_{ps3}}{T_{ps3}}x_{16} - \frac{K_{ps3}}{T_{ps3}}x_{17} - \frac{K_{ps3}}{T_{ps3}}w_3 \quad (7)$$

$$\dot{x}_8 = -\frac{1}{T_{t3}}x_8 + \frac{1}{T_{t3}}x_9 \quad (8)$$

$$\dot{x}_9 = -\frac{1}{R_3T_{sg3}}x_7 - \frac{1}{T_{sg3}}x_9 - \frac{1}{T_{sg3}}x_{14} + \frac{1}{T_{sg3}}x_{16} + \frac{1}{T_{sg3}}x_{17} \quad (9)$$

$$\dot{x}_{10} = -\frac{1}{T_{ps4}}x_{10} + \frac{K_{ps4}}{T_{ps4}}x_{11} - \frac{K_{ps4}}{T_{ps4}}x_{15} + \frac{a_{34}K_{ps4}}{T_{ps4}}x_{16} - \frac{K_{ps4}}{T_{ps4}}x_{18} - \frac{K_{ps4}}{T_{ps4}}w_4 \quad (10)$$

$$\dot{x}_{11} = -\frac{1}{T_{t4}}x_{11} + \frac{1}{T_{t4}}x_{12} \quad (11)$$

$$\dot{x}_{12} = -\frac{1}{R_4 T_{sg4}} x_{10} - \frac{1}{T_{sg4}} x_{12} + \frac{1}{T_{sg4}} x_{15} - \frac{a_{34}}{T_{sg4}} x_{16} + \frac{1}{T_{sg4}} x_{18} \quad (12)$$

$$\dot{x}_{13} = 2\pi T_{12} x_1 - 2\pi T_{12} x_4 \quad (13)$$

$$\dot{x}_{14} = -2\pi T_{31} x_1 + 2\pi T_{31} x_7 \quad (14)$$

$$\dot{x}_{15} = -2\pi T_{41} x_1 + 2\pi T_{41} x_{10} \quad (15)$$

$$\dot{x}_{16} = 2\pi T_{34} x_7 - 2\pi T_{34} x_{10} \quad (16)$$

$$\dot{x}_{17} = -2\pi T_{32} x_4 + 2\pi T_{32} x_7 \quad (17)$$

$$\dot{x}_{18} = -2\pi T_{42} x_4 + 2\pi T_{42} x_{10} \quad (18)$$

These eighteen equations can be organized in the following vector matrix form:

$$\dot{x} = Ax + Fw$$

Where, A is system matrix and F is disturbance distribution matrix,

$x = [x_1 \ . \ . \ . \ x_{18}]^T$ = state vector and

$\Delta P_d = w = [w_1 \ \ w_2 \ \ w_3 \ \ w_4]^T$ =disturbance vector.

Using equations $\dot{x}_1, \dot{x}_2, \ldots, \dot{x}_{18}$, matrix A is formulated, as given.

$$A = \begin{bmatrix}
\frac{-1}{T_{ps1}} & \frac{K_{ps1}}{T_{ps1}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{-K_{ps1}}{T_{ps1}} & \frac{a_{31}K_{ps1}}{T_{ps1}} & \frac{a_{41}K_{ps1}}{T_{ps1}} & 0 & 0 & 0 \\
0 & \frac{-1}{T_{t1}} & \frac{1}{T_{t1}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{-1}{R_1 T_{sg1}} & 0 & \frac{-1}{T_{sg1}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{T_{sg1}} & -\frac{a_{31}}{T_{sg1}} & -\frac{a_{41}}{T_{sg1}} & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{-1}{T_{ps2}} & \frac{K_{ps2}}{T_{ps2}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{a_{12}K_{ps2}}{T_{ps2}} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{-1}{T_{t2}} & \frac{1}{T_{t2}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{-1}{R_2 T_{sg2}} & 0 & \frac{-1}{T_{sg2}} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{-a_{12}}{T_{sg2}} & 0 & 0 & 0 & -\frac{a_{32}}{T_{sg2}} & -\frac{a_{42}}{T_{sg2}} \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{-1}{T_{ps3}} & \frac{K_{ps3}}{T_{ps3}} & 0 & 0 & 0 & 0 & 0 & \frac{-K_{ps3}}{T_{ps3}} & 0 & \frac{-K_{ps3}}{T_{ps3}} & \frac{-K_{ps3}}{T_{ps3}} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{-1}{T_{t3}} & \frac{1}{T_{t3}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{-1}{R_3 T_{sg3}} & 0 & \frac{-1}{T_{sg3}} & 0 & 0 & 0 & 0 & \frac{1}{T_{sg3}} & 0 & \frac{1}{T_{sg3}} & \frac{1}{T_{sg3}} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{-1}{T_{ps4}} & \frac{K_{ps4}}{T_{ps4}} & 0 & 0 & 0 & \frac{-K_{ps4}}{T_{ps4}} & \frac{a_{34}K_{ps4}}{T_{ps4}} & 0 & \frac{-K_{ps4}}{T_{ps4}} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{-1}{Tt_4} & \frac{1}{Tt_4} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{-1}{R_4 T_{sg4}} & 0 & \frac{-1}{T_{sg4}} & 0 & 0 & \frac{1}{T_{sg4}} & -\frac{a_{34}}{T_{sg4}} & 0 & \frac{1}{T_{sg4}} \\
2\pi T_{12} & 0 & 0 & -2\pi T_{12} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-2\pi T_{31} & 0 & 0 & 0 & 0 & 0 & 2\pi T_{31} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-2\pi T_{41} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2\pi T_{41} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 2\pi T_{34} & 0 & 0 & -2\pi T_{34} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -2\pi T_{32} & 0 & 0 & 2\pi T_{32} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -2\pi T_{42} & 0 & 0 & 0 & 0 & 0 & 2\pi T_{42} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}$$

Disturbance matrix F is given as:

$$F^T = \begin{bmatrix}
-\frac{K_{ps1}}{T_{ps1}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -\frac{K_{ps2}}{T_{ps2}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -\frac{K_{ps3}}{T_{ps3}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{K_{ps4}}{T_{ps4}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}$$

Considering Δf_1, Δf_2, Δf_3 and Δf_4 (i.e. change in frequency) as system's output variables, the output matrix can be given as:

$$y = Cx + Dw$$

Where, D=0 and C & x are given as:

$$C = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}$$

$$x^T = [x_1 \ \ \square \ \ \square \ \ \square \ \ x_{18}]$$

The Matlab commands [y,x]=step(A,Fw,C,D,1,t) and plot(t,y) are used to see the dynamics of Δf_1, Δf_2, Δf_3 and Δf_4 of four-area LFC controller for change in demand in w_1 (ΔP_{d1}). The results are verified through Simulink.

IV. INTEGRAL CONTROLLER FOR FOUR-AREA SYSTEM

The block diagram of four-area system with integral controller is given in Fig. 2.

Mathematical Model Development

The following modifications and additions will take place in the model developed without controller:

$$\dot{x}_3 = -\frac{1}{R_1 T_{sg1}} x_1 - \frac{1}{T_{sg1}} x_3 + \frac{1}{T_{sg1}} u_1 \quad (19)$$

$$\dot{x}_6 = -\frac{1}{R_2 T_{sg2}} x_4 - \frac{1}{T_{sg2}} x_6 + \frac{1}{T_{sg2}} u_2 \quad (20)$$

$$\dot{x}_9 = -\frac{1}{R_3 T_{sg3}} x_7 - \frac{1}{T_{sg3}} x_9 + \frac{1}{T_{sg3}} u_3 \quad (21)$$

$$\dot{x}_{12} = -\frac{1}{R_4 T_{sg4}} x_{10} - \frac{1}{T_{sg4}} x_{12} + \frac{1}{T_{sg4}} u_4 \quad (22)$$

$$\dot{x}_{19} = b_1 x_1 + x_{13} - a_{31} x_{14} - a_{41} x_{15} \tag{23}$$

$$\dot{x}_{20} = b_2 x_4 - a_{12} x_{13} - a_{32} x_{17} - a_{42} x_{18} \tag{24}$$

$$\dot{x}_{21} = b_3 x_7 + x_{14} + x_{16} + x_{17} \tag{25}$$

$$\dot{x}_{22} = b_4 x_{10} + x_{15} - a_{34} x_{16} + x_{18} \tag{26}$$

These twenty two equations can also be organized in the matrix form as follows:

$$\dot{x} = Ax + Bu + Fw \tag{27}$$

Where,

B is the input matrix and u is control vector.

Further considering the following,

$$u_1 = -k_{i1} x_{19} = -k_{i1} \int ACE_1 dt = \Delta P_{C1}(s)$$

$$u_2 = -k_{i2} x_{20} = -k_{i2} \int ACE_2 dt = \Delta P_{C2}(s)$$

$$u_3 = -k_{i3} x_{21} = -k_{i3} \int ACE_3 dt = \Delta P_{C3}(s)$$

$$u_4 = -k_{i4} x_{22} = -k_{i4} \int ACE_4 dt = \Delta P_{C4}(s)$$

$$ACE_1 = \Delta P_{tie,\ 12} + \Delta P_{tie,\ 13} + \Delta P_{tie,\ 14} + b_1 \Delta f_1$$

$$ACE_2 = \Delta P_{tie,\ 21} + \Delta P_{tie,\ 23} + \Delta P_{tie,\ 24} + b_2 \Delta f_2$$

$$ACE_3 = \Delta P_{tie,\ 31} + \Delta P_{tie,\ 32} + \Delta P_{tie,\ 34} + b_3 \Delta f_3$$

$$ACE_4 = \Delta P_{tie,\ 41} + \Delta P_{tie,\ 42} + \Delta P_{tie,\ 43} + b_4 \Delta f_4$$

The state space model can be represented by the following equation:

$$\dot{x} = Ax + Fw \tag{28}$$

Where, the system matrix A can be formulated, as given on next page.

Considering Δf_1, Δf_2, Δf_3 and Δf_4 (i.e. change in frequency) and $\Delta P_{tie,\ 12}$, $\Delta P_{tie,\ 41}$ (change in tie line power) as system output variables, the output matrix can be given as:

$$y = Cx + Dw \tag{29}$$

D=0 and C & x are given as:

$$C = \begin{bmatrix} 1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0 \end{bmatrix}$$

$$x^T = [x_1 \quad x_2 \quad x_3 \quad . \quad . \quad . \quad x_{21} \quad x_{22}]$$

The dynamic responses of Δf_1, Δf_2, Δf_3 and Δf_4 of four-area LFC controller for change in demand in w_1 (ΔP_{d1}) are obtained through Matlab program and the results are verified through Simulink.

V. OIPTIMAL INTEGRAL CONTROLLER FOR FOUR-AREA SYSTEM

Optimal control is a branch of modern control theory that deals with designing controls for dynamic systems by minimizing a performance index that depends on all system states. The object of optimal control regulator design is to determine the optimal control law w, which can transfer the system from its initial state to the final state so that a given performance index is minimized.

$$w = -Kx \tag{30}$$

Here K is feedback gain matrix.

A convenient performance index has the quadratic form of

$$J = \int_0^\infty (x^T Q x + w^T R w) dt \tag{31}$$

In order to design linear quadratic regulator the method of Lagrange multipliers, results in the following matrix Riccati equation

$$PA + A^T P + Q - PFR^{-1} F^T P = 0 \tag{32}$$

$$A = \begin{bmatrix}
-\frac{1}{T_{ps1}} & \frac{K_{ps1}}{T_{ps1}} & 0&0&0&0&0&0&0&0&0&0 & -\frac{K_{ps1}}{T_{ps1}} & \frac{a_{31}K_{ps1}}{T_{ps1}} & \frac{a_{41}K_{ps1}}{T_{ps1}} & 0&0&0&0&0&0&0 \\
0 & -\frac{1}{T_{t1}} & \frac{1}{T_{t1}} & 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
\frac{1}{R_1 T_{sg1}} & 0 & -\frac{1}{T_{sg1}} & 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 & -\frac{k_{i1}}{T_{sg1}} & 0&0&0 \\
0&0&0 & -\frac{1}{T_{ps2}} & \frac{K_{ps2}}{T_{ps2}} & 0&0&0&0&0&0&0 & \frac{a_{12}K_{ps2}}{T_{ps2}} & 0&0&0 & \frac{a_{32}K_{ps2}}{T_{ps2}} & \frac{a_{42}K_{ps2}}{T_{ps2}} & 0&0&0&0 \\
0&0&0&0 & -\frac{1}{T_{t2}} & \frac{1}{T_{t2}} & 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&0&0 & -\frac{1}{R_2 T_{sg2}} & 0 & -\frac{1}{T_{sg2}} & 0&0&0&0&0&0&0&0&0&0&0&0&0 & -\frac{k_{i2}}{T_{sg2}} & 0&0 \\
0&0&0&0&0&0 & -\frac{1}{T_{ps3}} & \frac{K_{ps3}}{T_{ps3}} & 0&0&0&0&0 & -\frac{K_{ps3}}{T_{ps3}} & 0 & -\frac{K_{ps3}}{T_{ps3}} & -\frac{K_{ps3}}{T_{ps3}} & 0&0&0&0&0 \\
0&0&0&0&0&0&0 & -\frac{1}{T_{t3}} & -\frac{1}{T_{t3}} & 0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&0&0&0&0&0 & -\frac{1}{R_3 T_{sg3}} & 0 & -\frac{1}{T_{sg3}} & 0&0&0&0&0&0&0&0&0&0&0 & -\frac{k_{i3}}{T_{sg3}} & 0 \\
0&0&0&0&0&0&0&0&0 & -\frac{1}{T_{ps4}} & \frac{K_{ps4}}{T_{ps4}} & 0&0&0 & -\frac{K_{ps4}}{T_{ps4}} & \frac{a_{34}K_{ps4}}{T_{ps4}} & 0 & -\frac{K_{ps4}}{T_{ps4}} & 0&0&0&0 \\
0&0&0&0&0&0&0&0&0&0 & -\frac{1}{T_{t4}} & \frac{1}{T_{t4}} & 0&0&0&0&0&0&0&0&0&0 \\
0&0&0&0&0&0&0&0&0 & -\frac{1}{R_4 T_{sg4}} & 0 & -\frac{1}{T_{sg4}} & 0&0&0&0&0&0&0&0&0 & -\frac{k_{i4}}{T_{sg4}} \\
2\pi T_{12} & 0&0 & -2\pi T_{12} & 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
-2\pi T_{31} & 0&0&0&0&0 & 2\pi T_{31} & 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
-2\pi T_{41} & 0&0&0&0&0&0&0&0 & 2\pi T_{41} & 0&0&0&0&0&0&0&0&0&0&0&0 \\
0&0&0&0&0&0 & 2\pi T_{34} & 0&0 & -2\pi T_{34} & 0&0&0&0&0&0&0&0&0&0&0&0 \\
0&0&0 & -2\pi T_{32} & 0&0 & 2\pi T_{32} & 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&0&0 & -2\pi T_{42} & 0&0&0&0&0 & 2\pi T_{42} & 0&0&0&0&0&0&0&0&0&0&0&0 \\
b_1 & 0&0&0&0&0&0&0&0&0&0&0 & 1 & -a_{31} & -a_{41} & 0&0&0&0&0&0&0 \\
0&0&0 & b_2 & 0&0&0&0&0&0&0&0 & -a_{12} & 0&0&0 & -a_{32} & -a_{42} & 0&0&0&0 \\
0&0&0&0&0&0 & b_3 & 0&0&0&0&0&0 & 1 & 0 & 1 & 1 & 0&0&0&0&0 \\
0&0&0&0&0&0&0&0&0 & b_4 & 0&0&0&0 & 1 & -a_{34} & 0 & 1 & 0&0&0&0
\end{bmatrix}$$

Where, the pair of matrices (A, F) and (A, C) are controllable and observable.

The solution of (32) yields a positive definite symmetric matrix P and the desired optimal feedback gain matrix is given by

$$K = R^{-1} F^T P \tag{33}$$

So the structure of optimal control law (30) is modifies as

$$w = -R^{-1}F^{T}Px \tag{34}$$

Using (28 and 30), A turns to A_f

$$A_f = A - FK \tag{35}$$

And (28) turns to

$$\dot{x} = A_f x \tag{36}$$

The Matlab control system toolbox function [K,P] = lqr2(A,F,Q,R) is used for the solution of matrix Riccati equation.

The Performance Index selected for four-area power system is of the following form:

$$J = \frac{1}{2}\int_0^{\infty}[(ACE_1)^2 + (ACE_2)^2 + (ACE_3)^2 + (ACE_4)^2 + ({x_{19}}^2 + {x_{20}}^2 + {x_{21}}^2 + {x_{22}}^2) + k({w_1}^2 + {w_2}^2 + {w_3}^2 + {w_4}^2)]dt \tag{37}$$

$$J = \frac{1}{2}\int_0^{\infty}[(b_1x_1 + x_{13} - a_{31}x_{14} - a_{41}x_{15})^2 + (b_2x_4 - a_{12}x_{13} - a_{32}x_{17} - a_{42}x_{18})^2 + (b_3x_7 + x_{16} + x_{14} + x_{17})^2 + (b_4x_{10} - a_{34}x_{16} + x_{18} + x_{15})^2 + ({x_{19}}^2 + {x_{20}}^2 + {x_{21}}^2 + {x_{22}}^2) + k({w_1}^2 + {w_2}^2 + {w_3}^2 + {w_4}^2)]dt \tag{38}$$

From J, matrices Q and R can be recognized as given.
Where, R (= KI) is Symmetric matrix.

$$Q = \begin{bmatrix}
{b_1}^2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & b_1 & -b_1a_{31} & -b_1a_{41} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & {b_2}^2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & b_2a_{12} & 0 & 0 & 0 & -b_2a_{32} & -b_2a_{42} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & {b_3}^2 & 0 & 0 & 0 & 0 & 0 & 0 & b_3 & 0 & b_3 & b_3 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & {b_4}^2 & 0 & 0 & 0 & 0 & b_4 & -b_4a_{34} & 0 & b_4 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
b_1 & 0 & 0 & b_2a_{12} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1+{a_{12}}^2 & -a_{31} & -a_{41} & 0 & a_{12}a_{32} & -a_{12}a_{42} & 0 & 0 & 0 & 0 \\
-b_1a_{31} & 0 & 0 & 0 & 0 & 0 & b_3 & 0 & 0 & 0 & 0 & 0 & -a_{31} & 1+{a_{31}}^2 & a_{31}a_{41} & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\
-b_1a_{41} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & b_4 & 0 & 0 & -a_{41} & a_{31}a_{41} & 1+{a_{41}}^2 & -a_{34} & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & b_3 & 0 & 0 & -b_4a_{34} & 0 & 0 & 0 & 1 & -a_{34} & 1+{a_{34}}^2 & 1 & -a_{34} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -b_2a_{32} & 0 & 0 & b_3 & 0 & 0 & 0 & 0 & 0 & a_{12}a_{32} & 1 & 0 & 1 & 1+{a_{32}}^2 & a_{32}a_{42} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -b_2a_{42} & 0 & 0 & 0 & 0 & 0 & b_4 & 0 & 0 & -a_{12}a_{42} & 0 & 1 & -a_{34} & a_{32}a_{42} & 1+{a_{42}}^2 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}$$

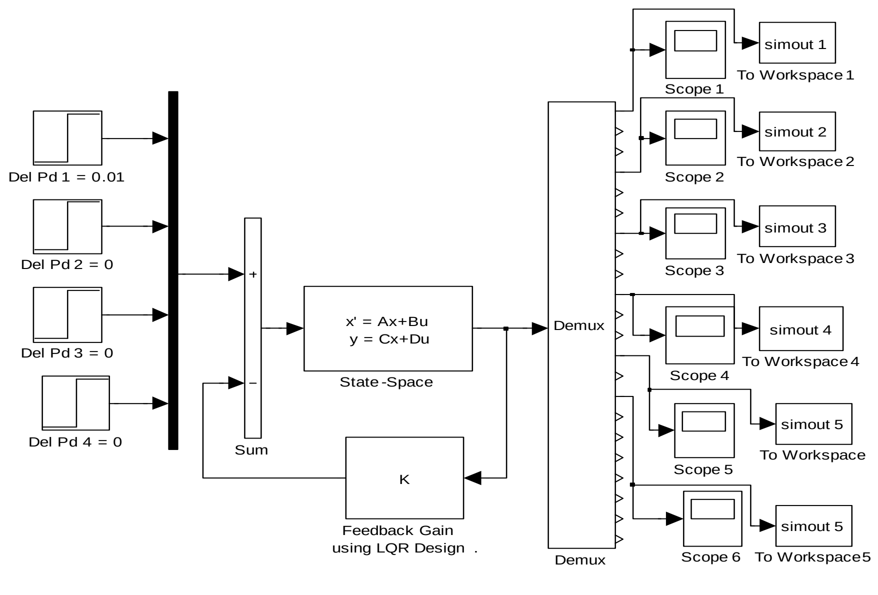


Fig. 3 Simulink model of optimal four-area system using LQR design

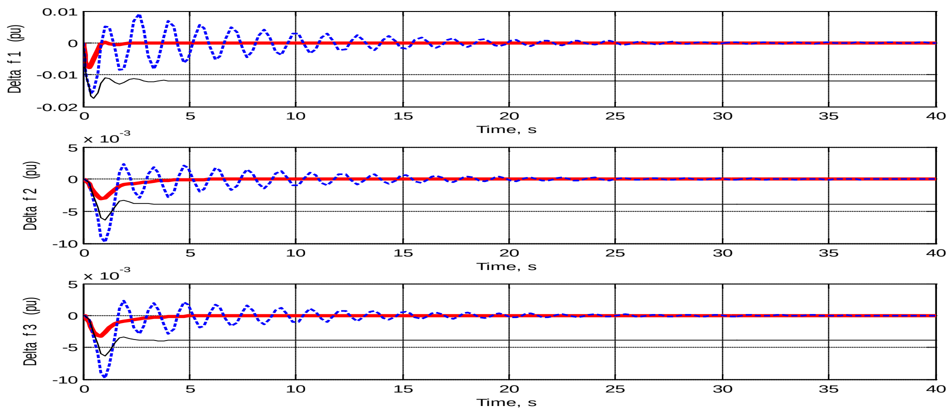
$$R = K\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

A Matlab program is written using matrices A, F, C, D, Q and R. The command[y, x] =step (A_f, Fw, C, D, 1, t) is used to get the system's response. A Matlab Simulink model given in Fig.3 can also be used to verify the system's response.

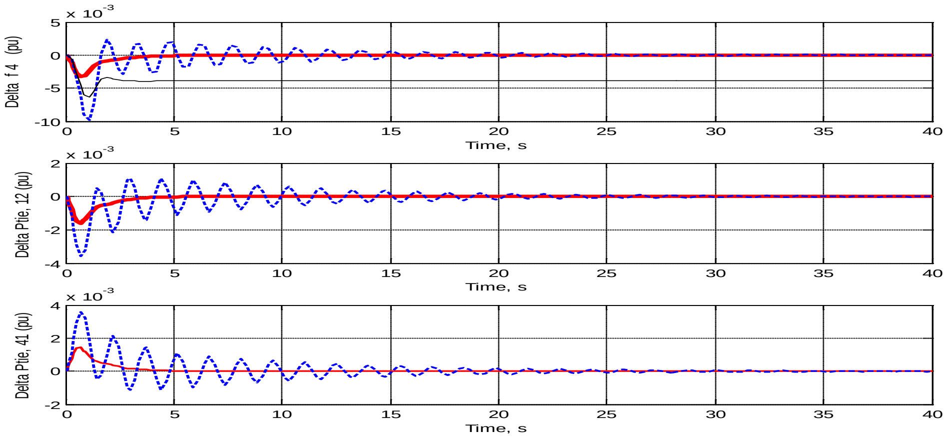
VI. RESULTS AND DISCUSSION

Simulations were performed without controller, with Integral and the proposed optimal integral controllers applied to a four-area interconnected power system as shown in Fig.2, and Fig.3 by applying 0.01 puMW step load disturbance to Area 1. The same system parameters are used as given in [2], [11], [12]. The implementation is done on Matlab7.5 Simulink-Workspace software. The simulations were run on a

personal computer Intel Core2Duo CPU T5450 @ 1.66 GHz, 982 MHz, 2GB of RAM, under Window XP. The frequency deviations of Area 1 to Area 4 and some tie line power deviations with 0.01 puMW step load disturbance in Area 1 are shown in Fig.4. The frequency deviation of Area 1 is presented in Fig.5 in a larger scale. The integral controller improves the system performance in comparison when no controller is used by eliminating steady state frequency errors. Moreover, the optimal integral controller is significantly superior to the integral controller. It gives a better performance than the integral controller. The settling time and peak overshoots are reduced considerably as shown in Fig.5.



(a)



(b)

Fig. 4 Deviation in frequencies of (a) area 1, 2 and 3 (b) area 4, deviation in tie line power, $\Delta P_{tie,\,12}$ and $\Delta P_{tie,\,41}$ for 1% change in load (ΔP_{d1}=0.01 pu).

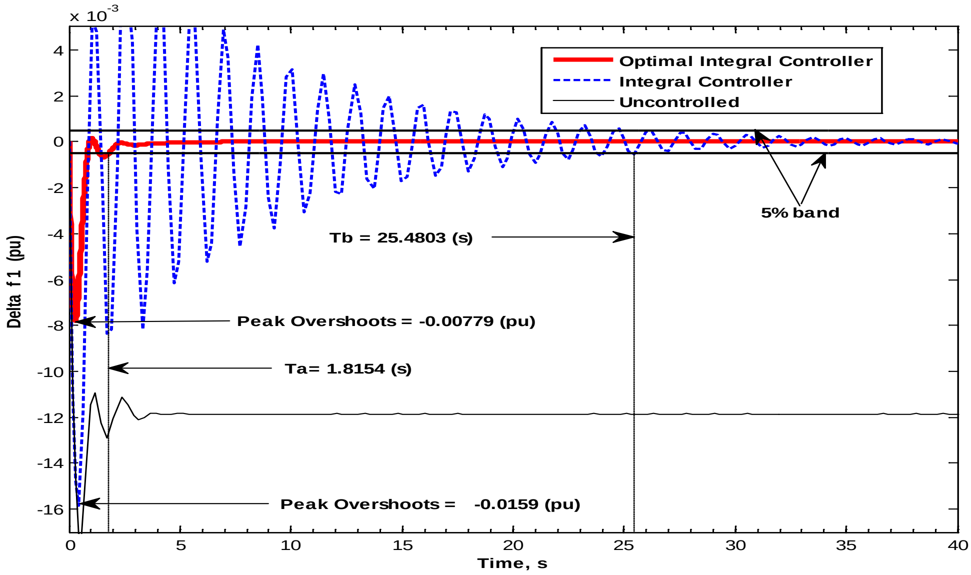


Fig. 5 The frequency deviation of the first area in larger scale indicating peak overshoot and settling time for integral and optimal Integral controller for 1% change in load (ΔP_{d1}=0.01 pu).

The frequency deviation of Area1 with settling time for 5% band of the step load change and maximum overshoots is given in Table I. The comparison of dynamic performances of various controllers with the proposed optimal integral controller shows that the proposed controller gives better results in terms of lesser settling time and peak overshoot. The simulation was repeated with various instantaneous load changes and it was found that results from the controller are better every time. The simulation results show that proposed controller for load frequency control is given 47.68% reduction in settling time and 59.81% reduction in peak overshoots when compared with Cam and Kocaarslan study (for two-area with same data).

TABLE I SYSTEM PERFORMANCE OF VARIOUS CONTROLLERS

Deviation in frequency (Δf_1)	**Settling time (s) for 5% band of step change**	**Peak over-shoot (pu)**
Proposed study (Ta)	1.8154	-0.00779
Cam's study (for two-area with FGPI) [11]	3.47	-0.0194
Mathur's study (for two-area with fuzzy controller) [10]	3.53	-0.0202
Oysal's study (with NN) [15]	6.81	-0.0149
Ghoshal's study (for three-area with Fuzzy Controller) [9]	8.0	-0.028
Integral Controller (Tb)	25.4803	-0.0159

In the analysis of the simulation results, the frequency deviation results were also used to calculate the integral absolute error (IAE) as given in Equation (39) for 40s of simulation time.

$$IAE = \int_0^{40} |\Delta f_1| \, dt \tag{39}$$

TABLE II SYSTEM PERFORMANCE IN TERMS OF IAE

Controllers	**IAE for Δf_1**
Optimal Integral Controller	0.004663
Integral Controller	0.06361
Uncontrolled System	0.4757

System performance of the controllers in terms of integral absolute error for frequency deviation of area 1 given in Table Ⅱ also shows that proposed optimal integral controller has less integral absolute error as compared to conventional integral controller.

VII. CONCLUSION

In this paper, an optimal integral controller is designed for automatic load frequency control of interconnected four-area power system by the use of optimal control design technique. The controller performance is observed on the basis of dynamic parameters i.e. settling time, peak overshoot and IAE. From Table I, it is concluded that the proposed controller for four-area has better dynamic performance when compared with many recent trend controllers applied to two-area and three-area interconnected power systems.

APPENDIX

The System data used in the study is given as: P_{ri}= 2000 MW, P_{rj}= 2000 MW, T_{sgi} = 0.08 sec., T_{psi} = 20 sec., T_{ti} = 0.03 sec., R_i = 2.4 $H_Z$pu/MW, T_{ij} = 0.08674, a_{ij} = 1, K_{ii} = 0.671, K_{psi} = 120 $H_Z$pu/MW, b_i= 0.425 puMW/H_Z, ΔP_{d1}=0.01 puMW, i, j= 1, 2, 3, & 4

NOTATIONS

f_i nominal system frequency of i^{th} area, H_Z

Δf_i incremental frequency deviation of i^{th} area, H_Z pu

T_{sgi} speed governor time constant of i^{th} area, sec.

R_i speed governor regulation of the of i^{th} area, H_Z/puMW

T_{ti} turbine time constant of i^{th} area, sec.

Q positive semi-definite symmetric state cost weighting matrix

R positive definite symmetric control cost weighting matrix

k optimal feedback gain matrix

K_{psi} gain of power system (generator load) of i^{th} area, H_Z/puMW

T_{psi} power system time constant of i^{th} area, sec.

ΔP_{gi} incremental generator power output change of i^{th} area, puMW

ΔP_{ti} incremental turbine power output change of i^{th} area, puMW

ΔP_{ci} incremental speed changer setting change of i^{th} area, puMW

ΔY_{Ei} incremental steam valve setting change of i^{th} area, puMW

K_{ii} gain of integral controller of i^{th} area

b_i frequency bias of i^{th} area, puMW/H_Z

ACE_i area control error of i^{th} area, puMW

u_i control input of i^{th} area

ΔP_{di} incremental load demand change of i^{th} area, puMW

w_i disturbance vector of i^{th} area, puMW

$\Delta P_{tie, ij}$ incremental tie line power change of i^{th} and j^{th} area, puMW

a_{ij} area size ratio coefficient

T_{ij} tie line synchronizing power co-efficient (pu) between area i and area j

i & j subscript referred to area i & j. (i, j =1, 2, 3 & 4)

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